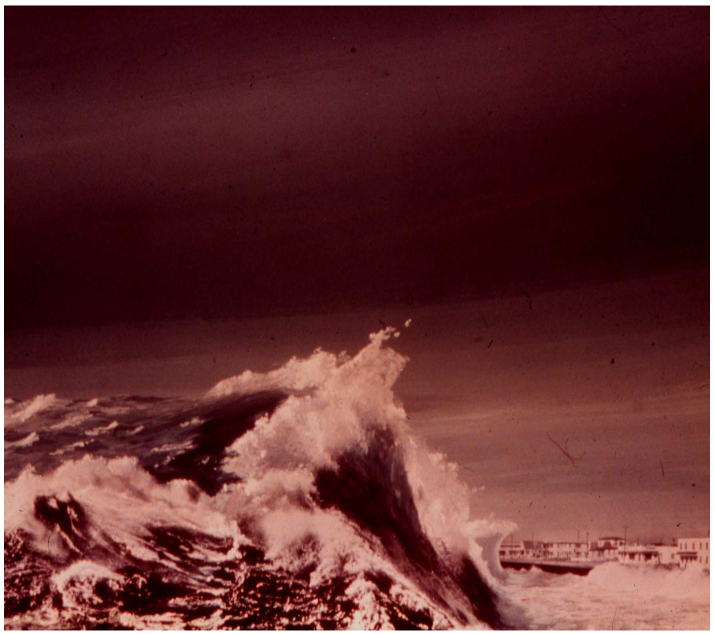
EVT, climate change & coastal flood risk



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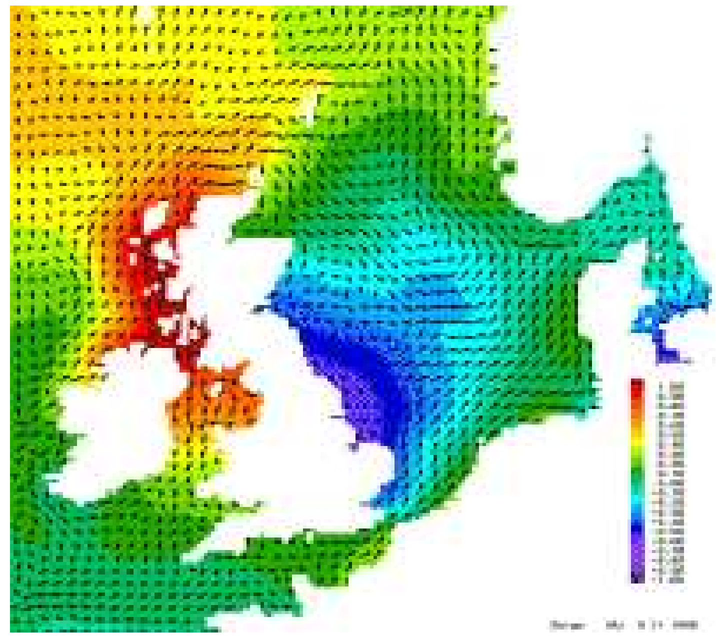

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Seminar at **RSS Highland local group**, October 2005

Adam Butler, Biomathematics & Statistics Scotland

Structure of the talk

1 : Storm surges

2 : Extreme value methods

3 : Our analysis

4 : Wider statistical context

1 : Storm surges

- **Sea levels**:

  Sea level = Still water level + Surface waves

  Still water level = Mean sea level + Tide + Surge

- **Surges**:

  Surge currents generated by wind stress & air pressure

  Distort sea levels, especially in shallow constrained basins

  Potentially major contribution to coastal flooding

- **Sea level change**:

  Increase in global mean sea levels

  Also local effects of isostatic readjustment

  But are there different trends in extreme sea levels... ?

- **Changes in the storm surge climate**:

  Increased storminess in the NE Atlantic (WASA, 1998)

  could increase magnitude and frequency of storm surges

  Little evidence for such a change (Bijl *et al.*, 1999)

- **Numerical storm surge models**:

  Complex mechanistic models to describe surge dynamics

  (Bode and Hardy, 1997)

- **Analysis of model output**:

  Use models to reconstruct past storm surge climates

  Use EVT to analyse statistical properties of model output

  (Flather *et al.*, 1998)

  We analyse temporal trends in these properties

- **Our 'model data'**:

  Output from a numerical storm surge model

  for the European Continental Shelf

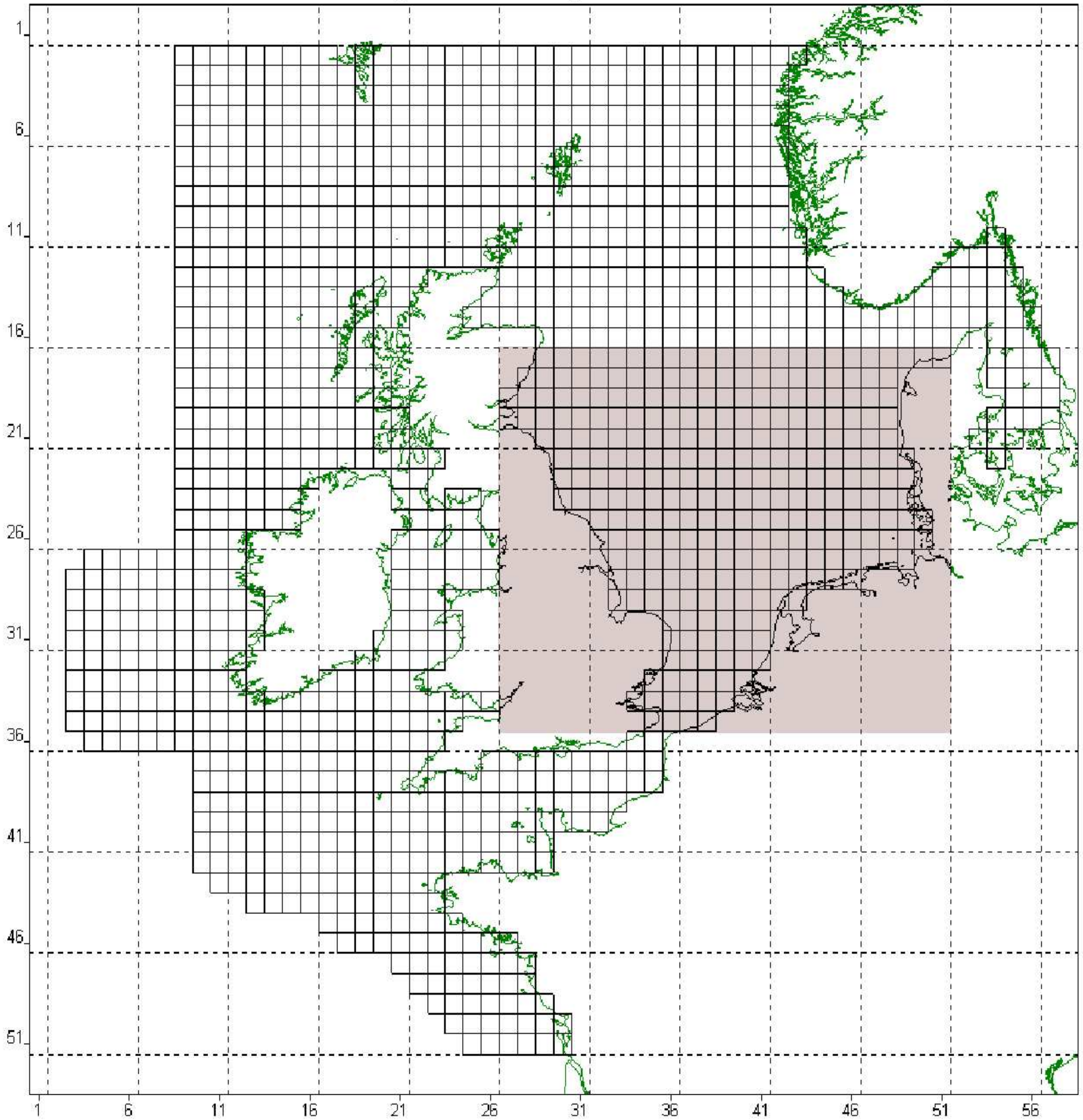
  Model forced using reconstructed meteorological data

  for the period 1955-2000

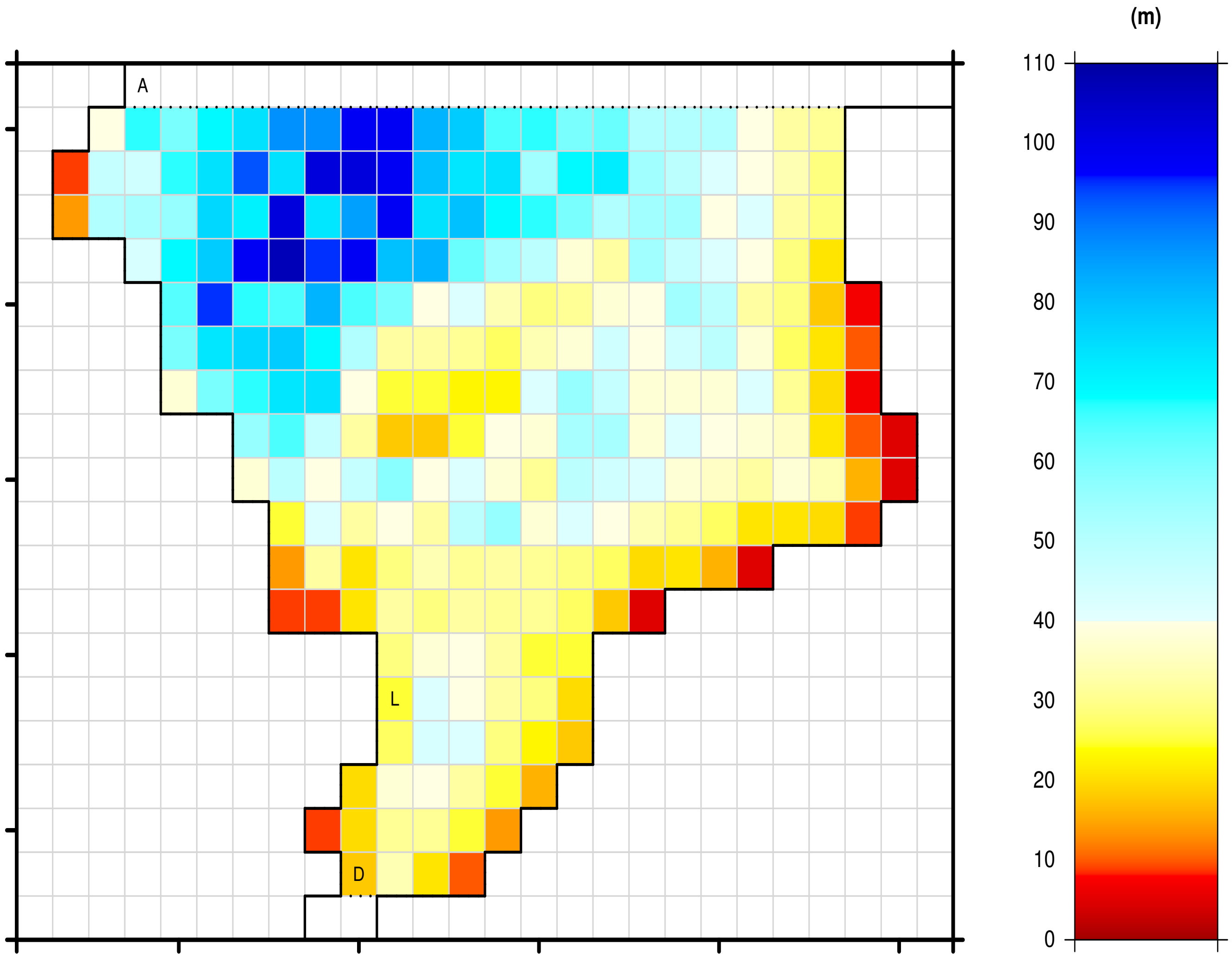
  Spatial resolution of approximately 35×35km

  403248 hourly surge elevations for each of 1928 grid cells

Grid for the CSX model



Water depth in the North Sea



2 : Extreme value methods

- **Typical extreme value problems**:

- Exceedance probabilities, $\mathbb{P}(X > u)$
  e.g. what is the probability that a sea wall of height u
  is overtopped during one year ?

- N-year return levels, $q_N = \{u : \mathbb{P}(X > u) = 1/N\}$
  e.g. what height should we build a sea wall such that it is
  overtopped with probability $1/N$ in a particular year ?

- **Common features**:

  We are interested in an event in the tail of the distribution

  There is little or no data available on such events -

  we are often extrapolating

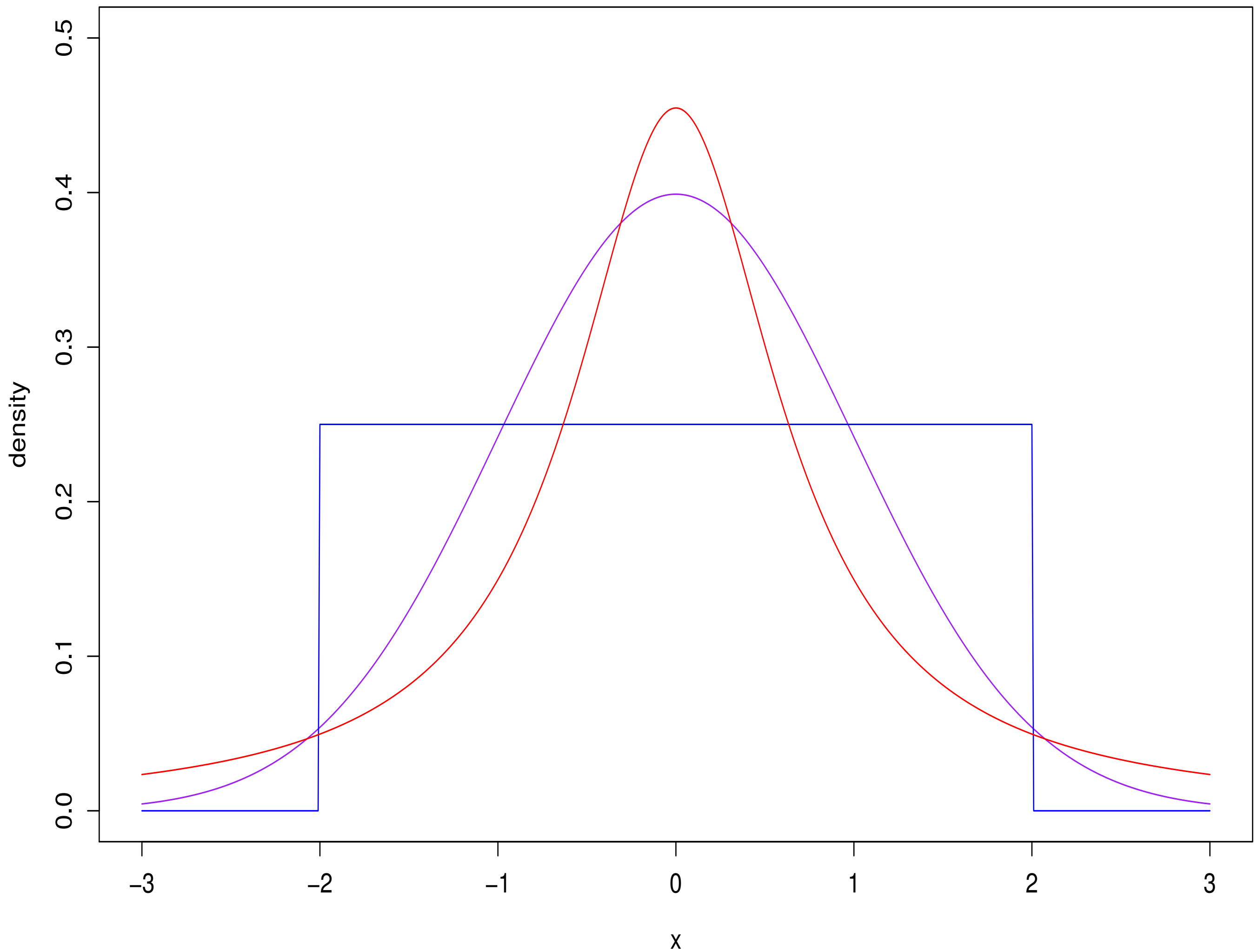
- **A worrying example**:

  The trouble with standard models is that inferences about the tails are *highly* sensitive to model mis-specification

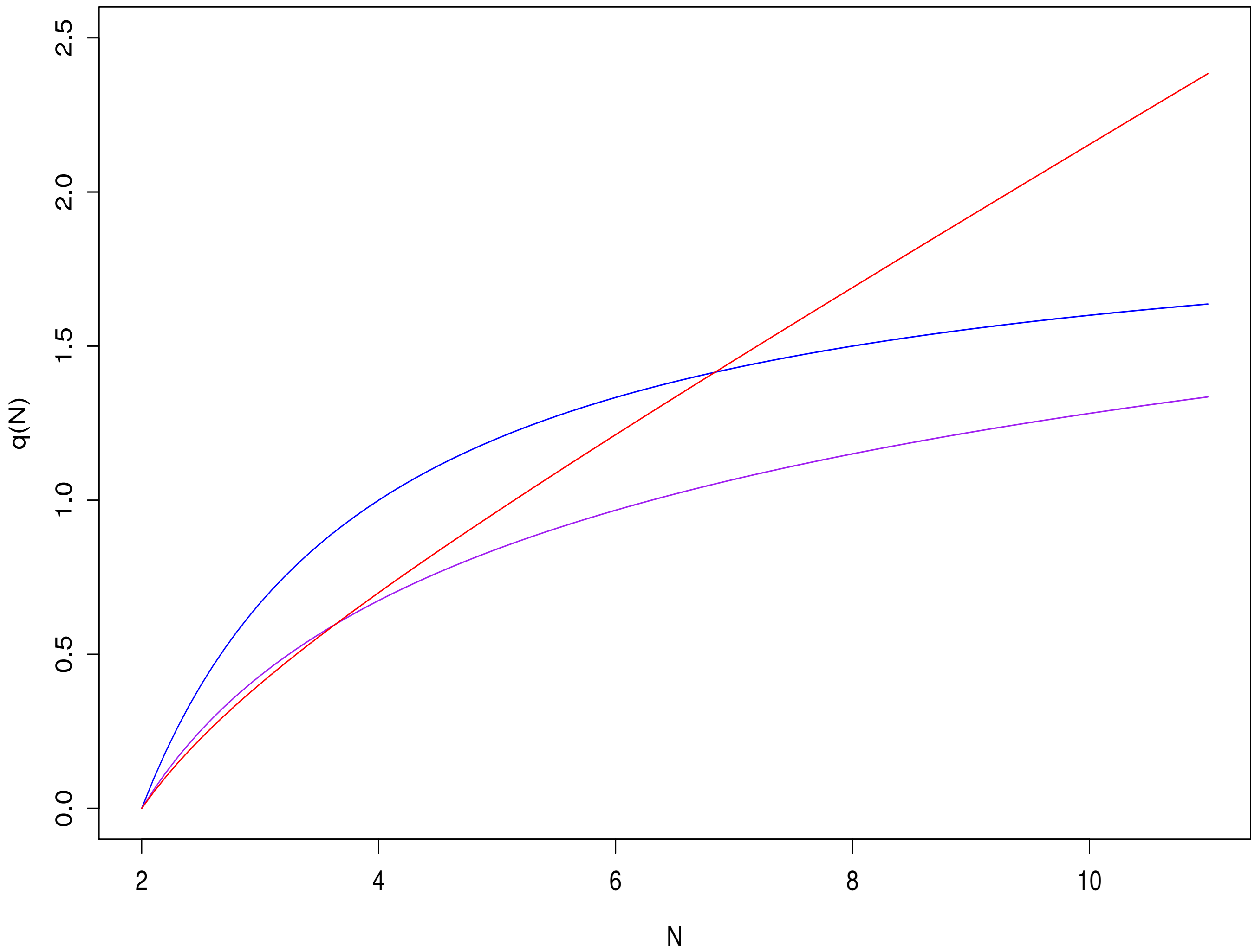
  e.g. consider the following distributions:

  Normal(0,1), Cauchy(0,0.7) and Uniform(-2,2)...

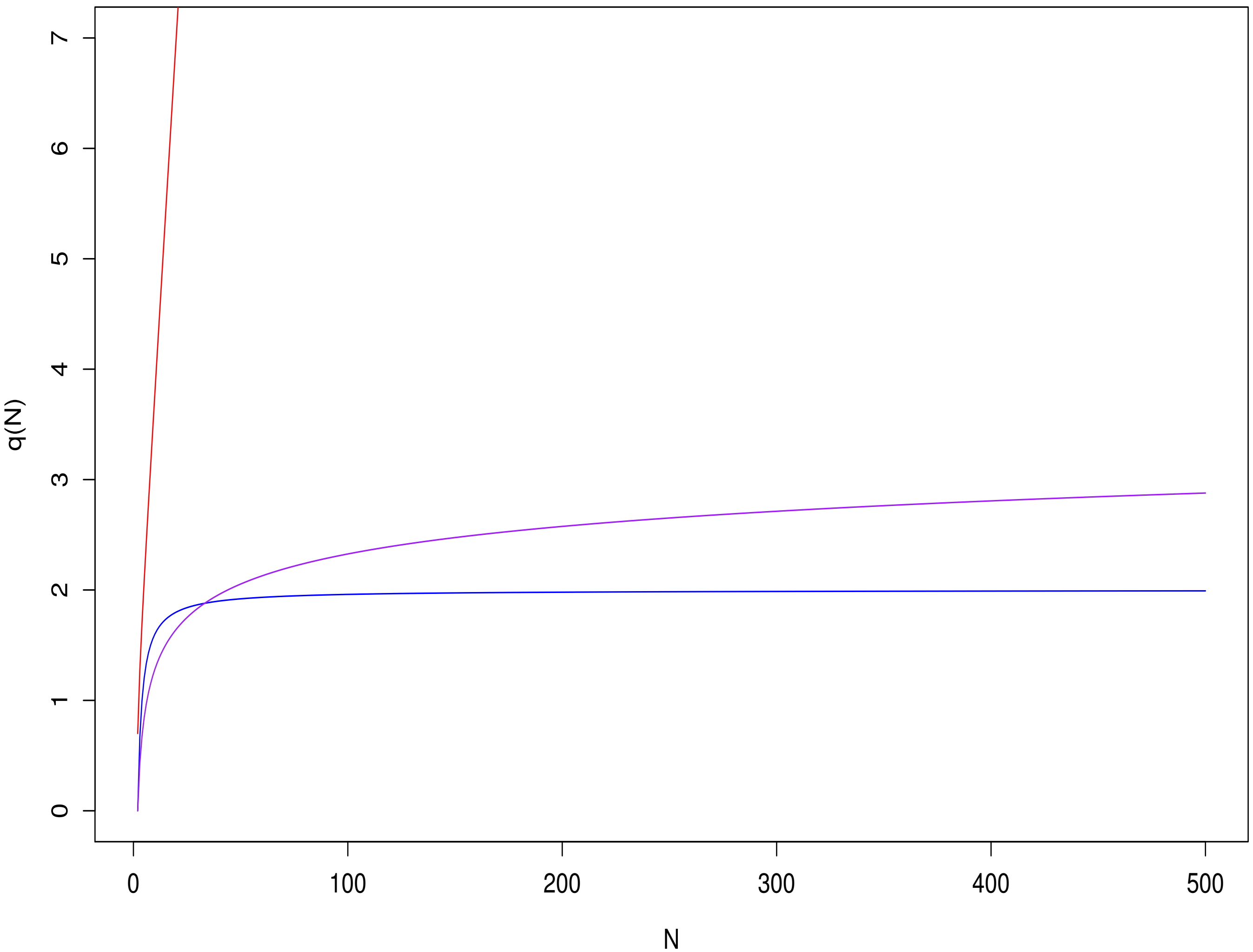
Density functions: Cauchy (red), Normal (purple), Uniform (blue)



Return levels: Cauchy (red), Normal (purple), Uniform (blue)



Return levels, again: Cauchy (red), Normal (purple), Uniform (blue)



- **The extreme value approach**:

  Base inferences about extreme events

  only on data at relatively extreme levels

  Models for extreme data motivated by asymptotic theory

  Provides a relatively robust basis for extrapolation

- **Key assumptions**:

  We are looking at a sufficiently extreme level

  Extreme events are drawn from a common population

  The data are free from outliers

- **Threshold exceedance approach**:

  Describes exceedances of a sufficiently high threshold u

  As $u \longrightarrow \infty$ the conditional distribution $(X - u)|X > u$ tends to a generalised Pareto distribution,

$$F(z) = 1 - \left(1 + \frac{\xi z}{\phi}\right)^{-1/\xi},$$

  with scale parameter ϕ and shape parameter ξ

- **Block maxima approach approach**:

  Describes the maxima of sufficiently long blocks

  As $n \longrightarrow \infty$ the (appropriately normalised) maxima $\max(X_1, \ldots, X_n)$ tend to a GEV distribution

$$F(z) = \exp\left[-\left\{1 + \xi\left(\frac{z-\mu}{\sigma}\right)\right\}^{-1/\xi}\right],$$

  with location parameter μ, scale parameter σ & shape parameter ξ.

- **r-largest value approach**:

  Describes the r largest events within a block (e.g. year)

  Generalisation of the GEV model,

  with the same parameters μ, σ and ξ

  Makes more efficient use of the available data

  Asymptotic motivation relies upon r being sufficiently small

- **Statistical inference & modelling**:

  Assume asymptotic models are valid at some finite level

  Selection of an appropriate level (u, n or r) is *difficult*
  but parameter stability plots are a useful tool

  Estimate model parameters via e.g. maximum likelihood

  Thereby estimate return levels & exceedance probabilities

- **Resources**:

  Introductory book: Coles, S.G. (2001) An Introduction to Statistical Modelling of Extreme Values, Springer.

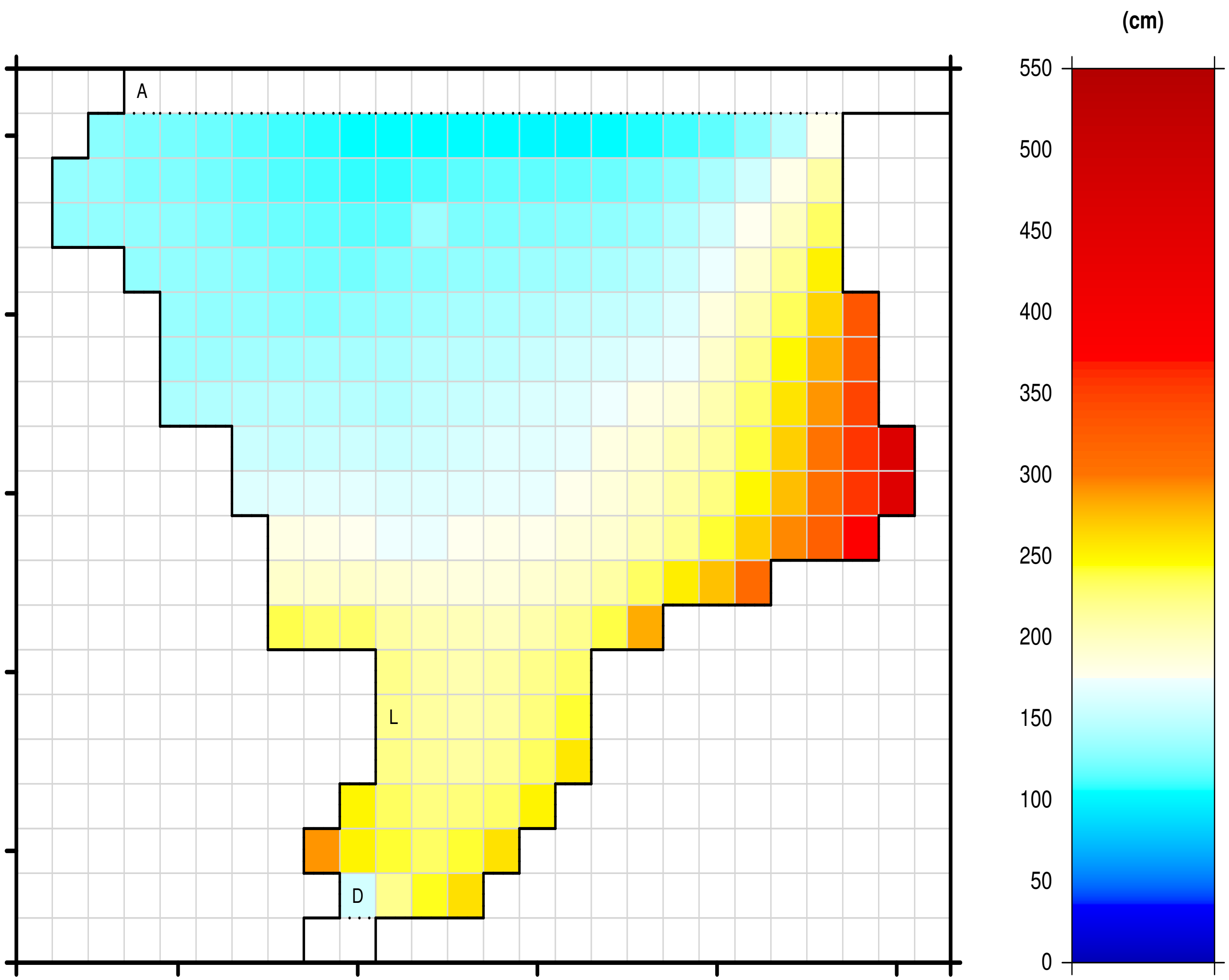
  Software:

  www.maths.lancs.ac.uk/ stephena/software.html

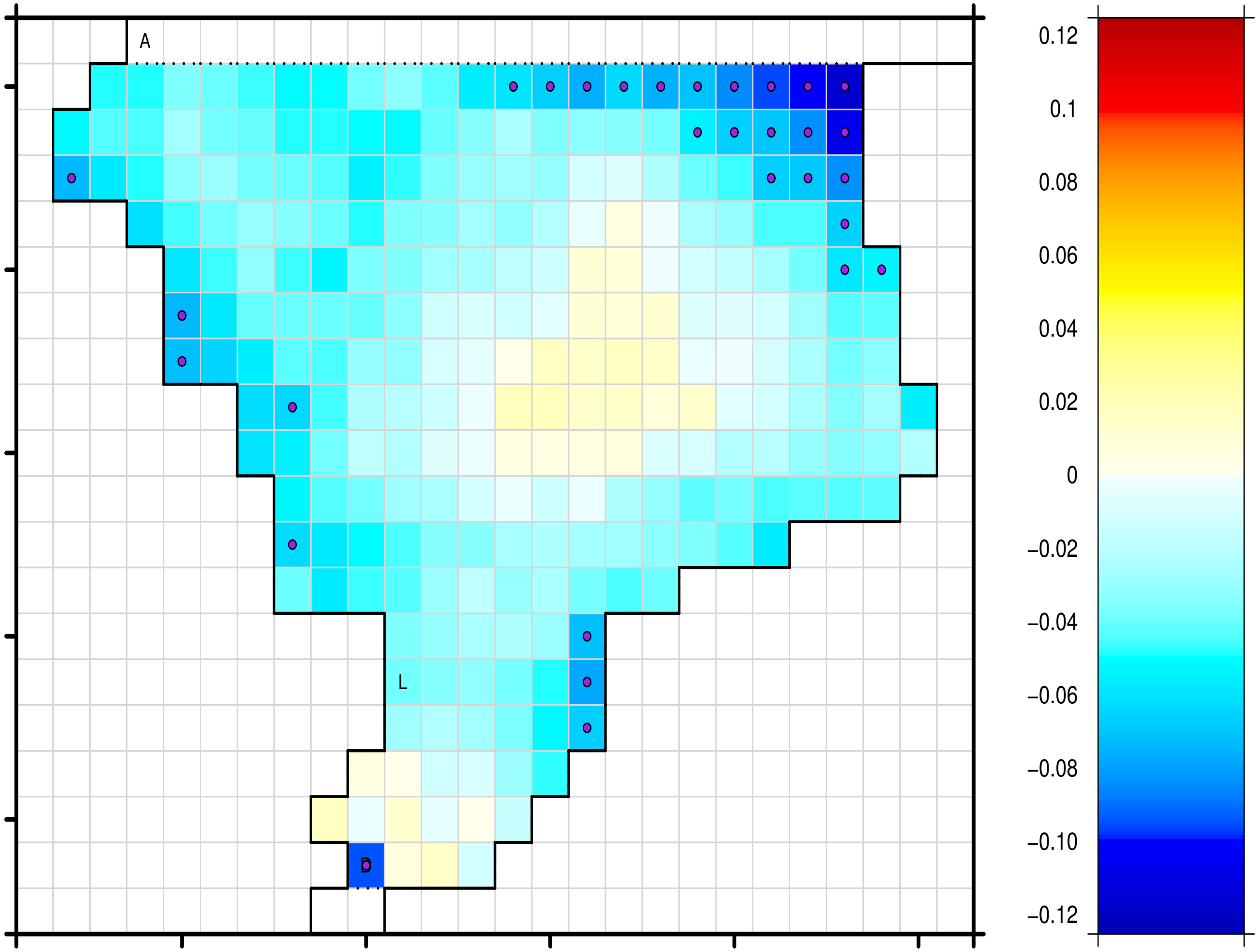
  Web resources:

  www.cru.uea.ac.uk/cru/projects/mice/html/extremes.html

Estimated 50y return levels, based on an r-largest fit at each site



Shape parameter estimates, based on an r-largest fit at each site



3 : Our analysis

- Let $X_{ij}^{(k)}$ = k-th largest surge elevation in year t_j at site s_i
- Assume that $(X_{ij}^{(1)}, \ldots, X_{ij}^{(r)})$ has an r-largest model with parameters $\mu(\mathbf{s}_i, t_j)$, $\sigma(\mathbf{s}_i, t_j)$ and $\xi(\mathbf{s}_i, t_j)$
- Model the parameters as smooth functions of time & space
- This is a form of nonparametric regression

- **The model: temporal smoothing only**

  Follow the approach of Davison and Ramesh (2000)

  For year t_j at site s_i maximise the local likelihood

$$\sum_{J=1}^{n} w_{Jj} l \left[(\mu_{ij}, \sigma_{ij}, \xi_{ij}); X_{iJ}^{(1)}, \ldots, X_{iJ}^{(r)} \right]$$

  The weights w_{Jj} determine the degree of smoothing

- **Statistical issues**

  Choice of weight function - e.g. Gaussian kernel:

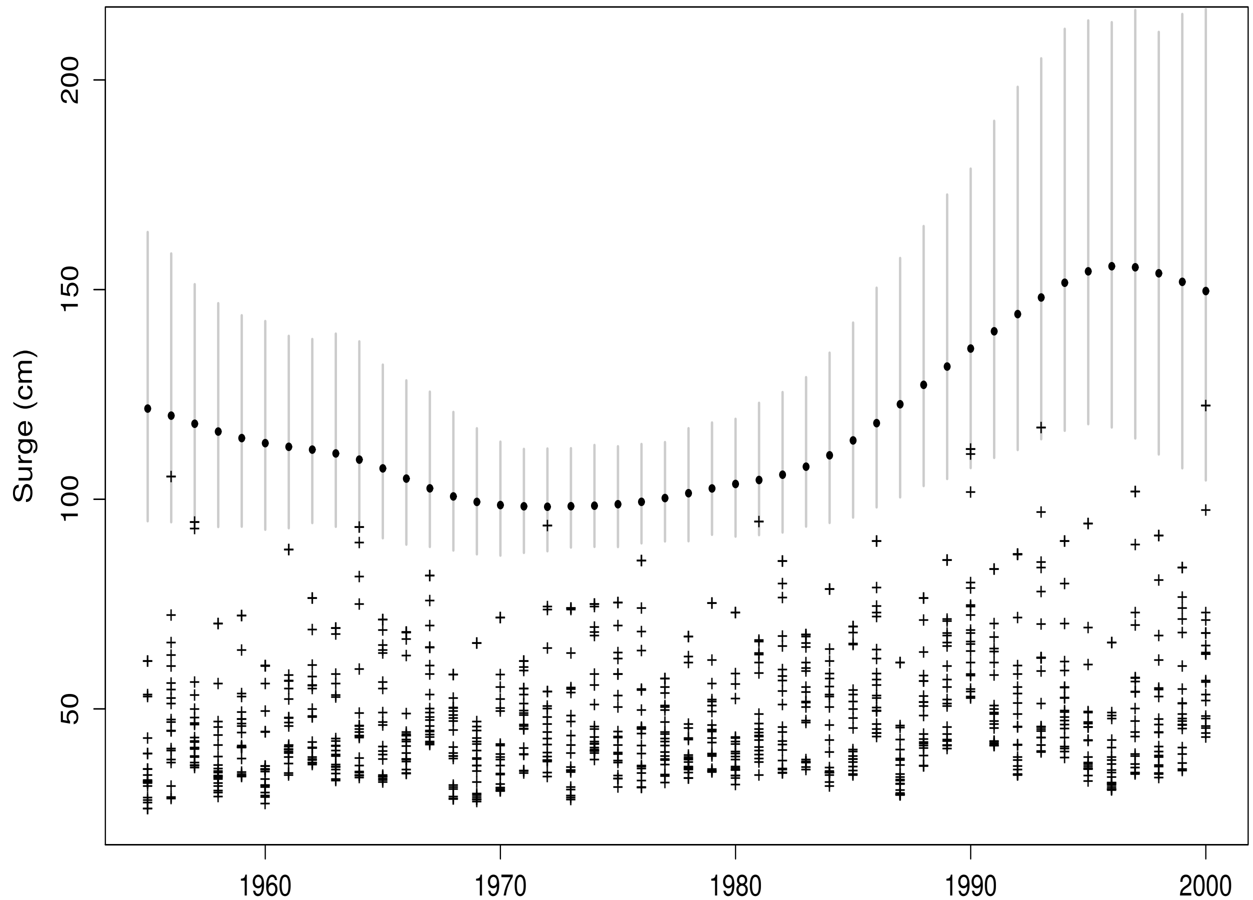
$$w_{Jj} = \phi\left(\frac{t_J - t_j}{h}\right)$$

  Select the bandwidth h either:

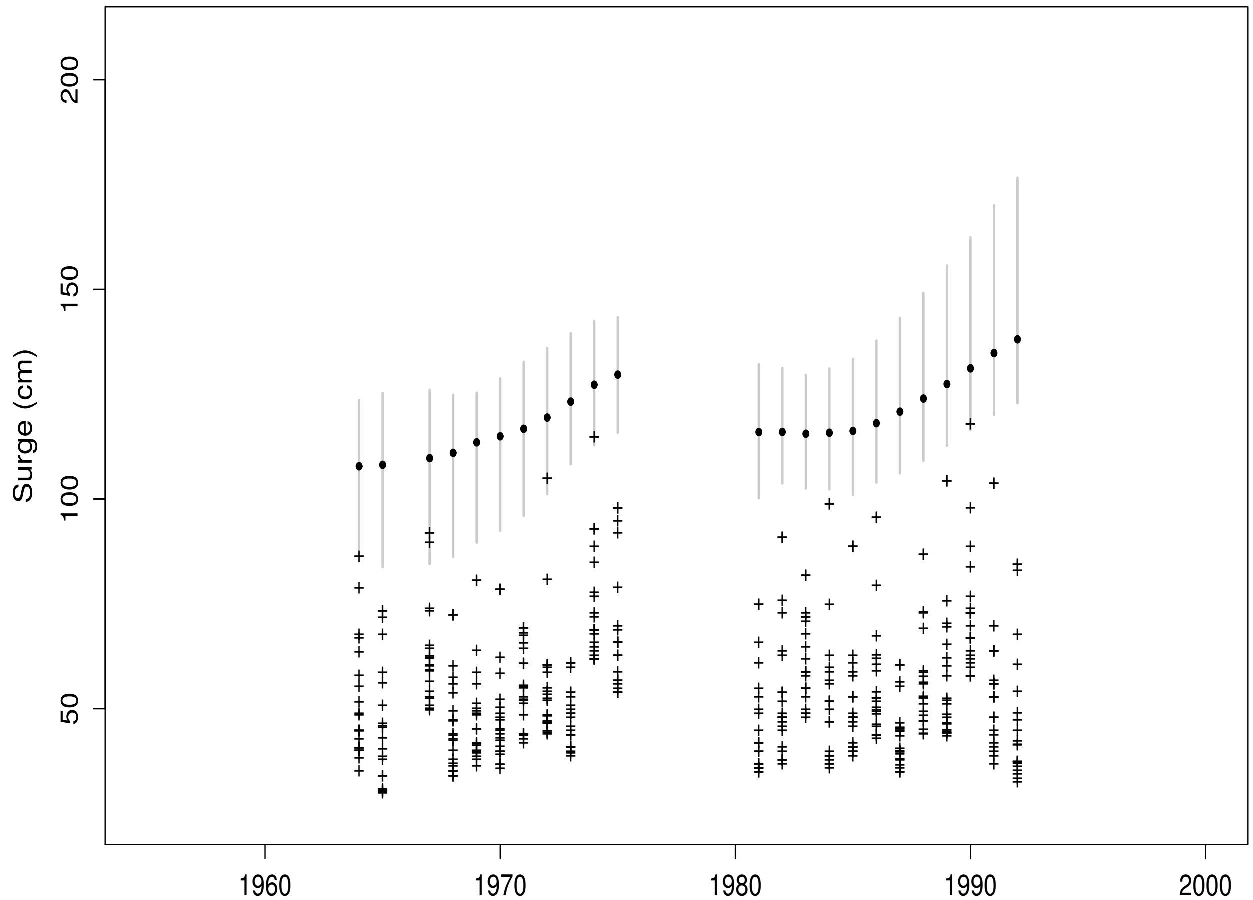
  - subjectively, based on scientific knowledge; or
  - using an automatic criterion e.g. cross-validation

  Assess model fit e.g. using a form of pooled Q-Q plot

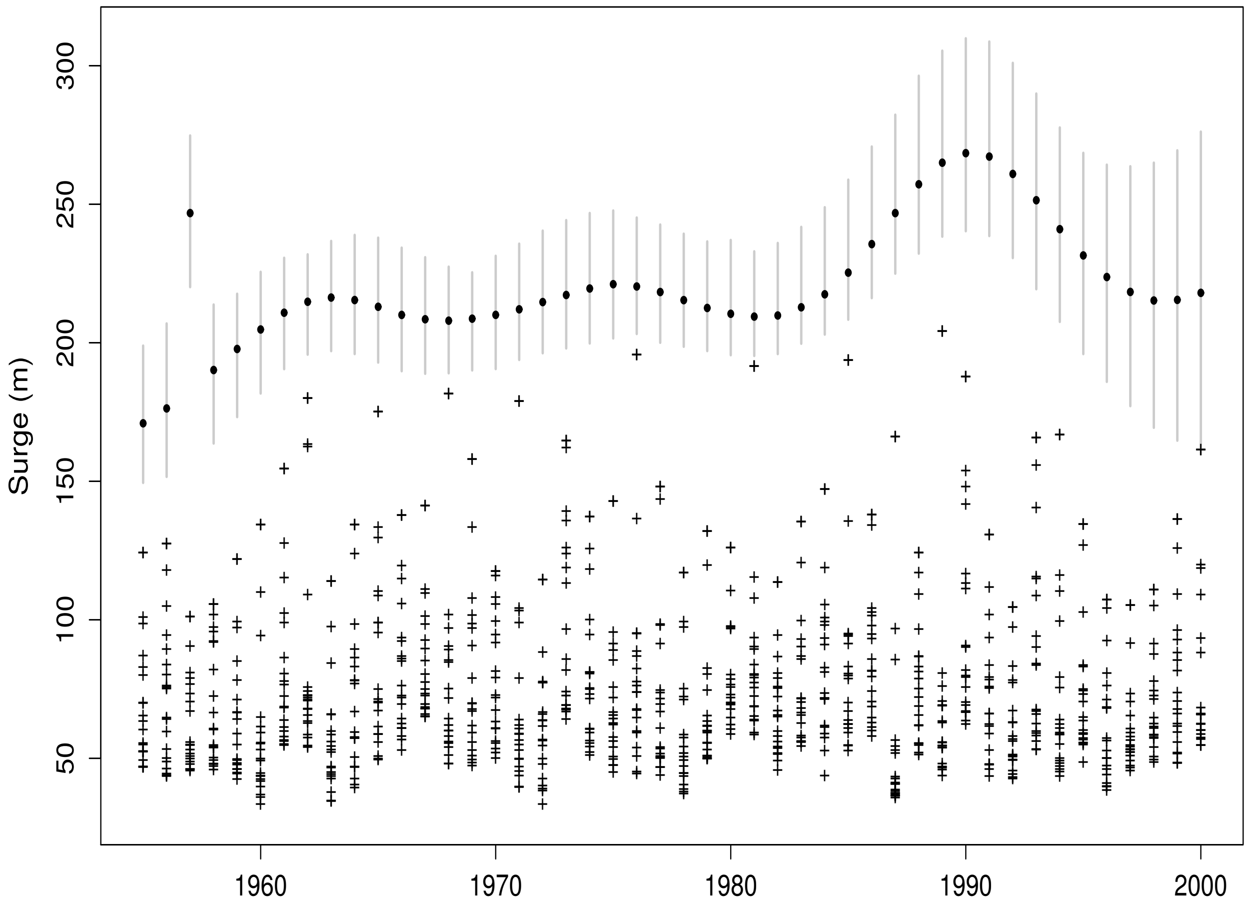
50y surge return levels: model output for Aberdeen



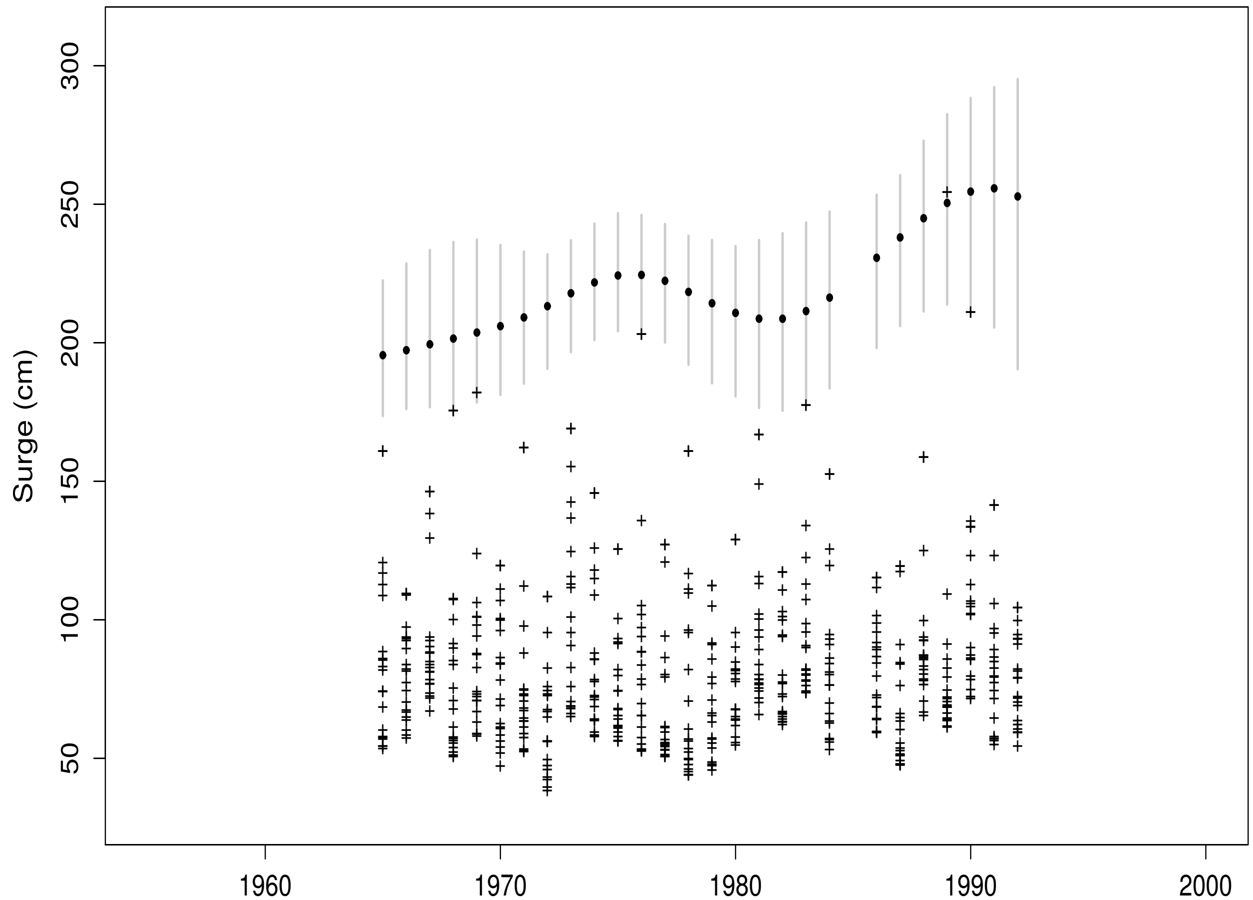
50y surge return levels: observational data for Aberdeen



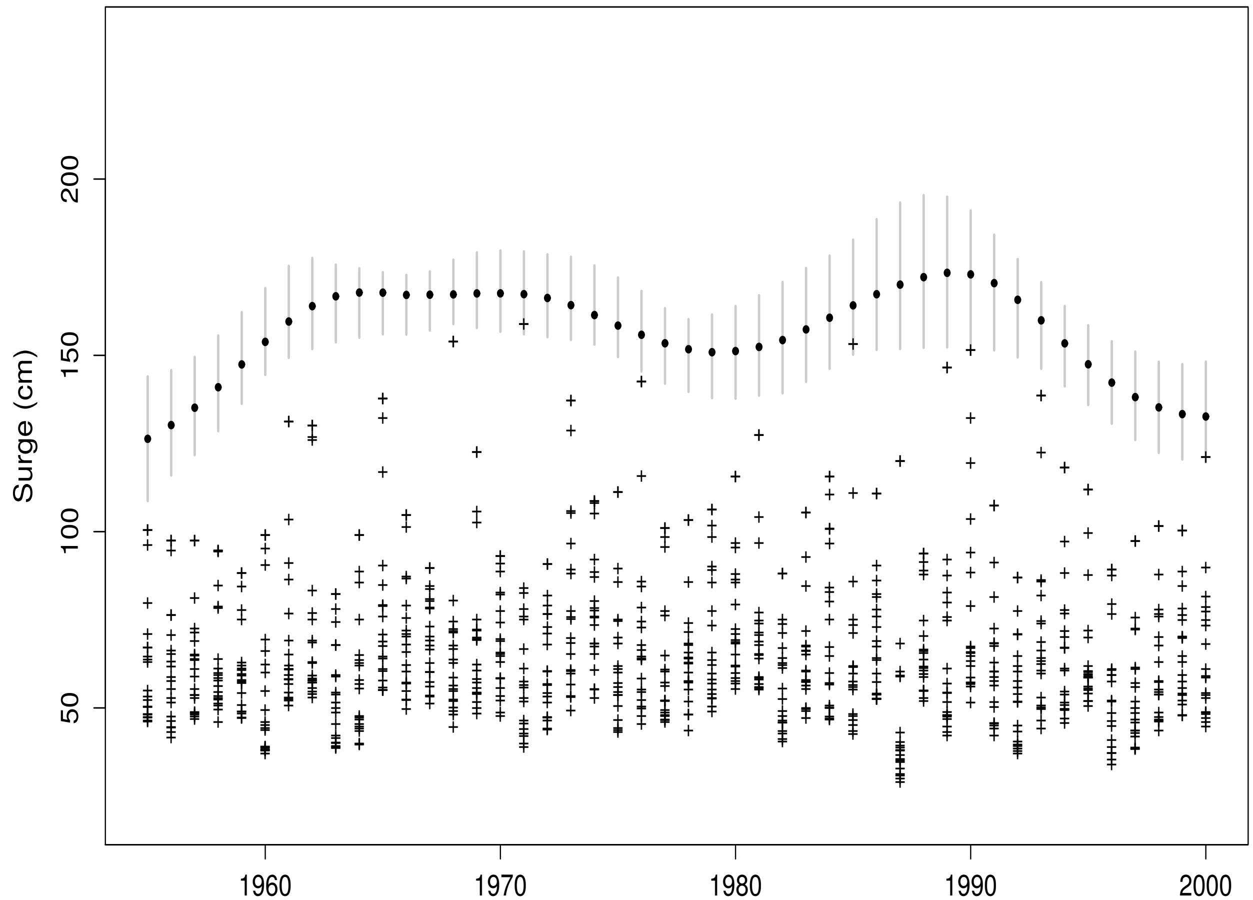
50y surge return levels: model output for Lowestoft



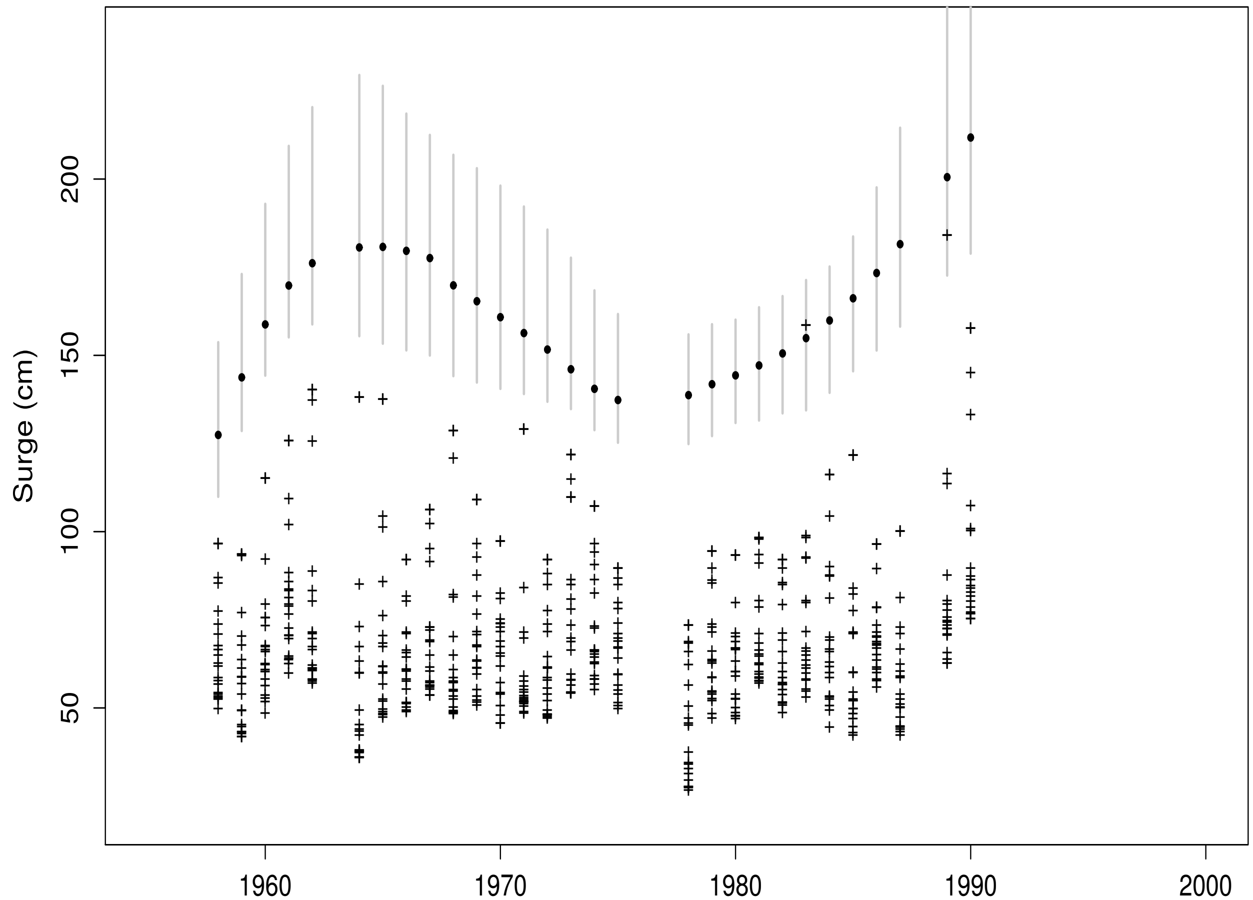
50y surge return levels: observational data for Lowestoft



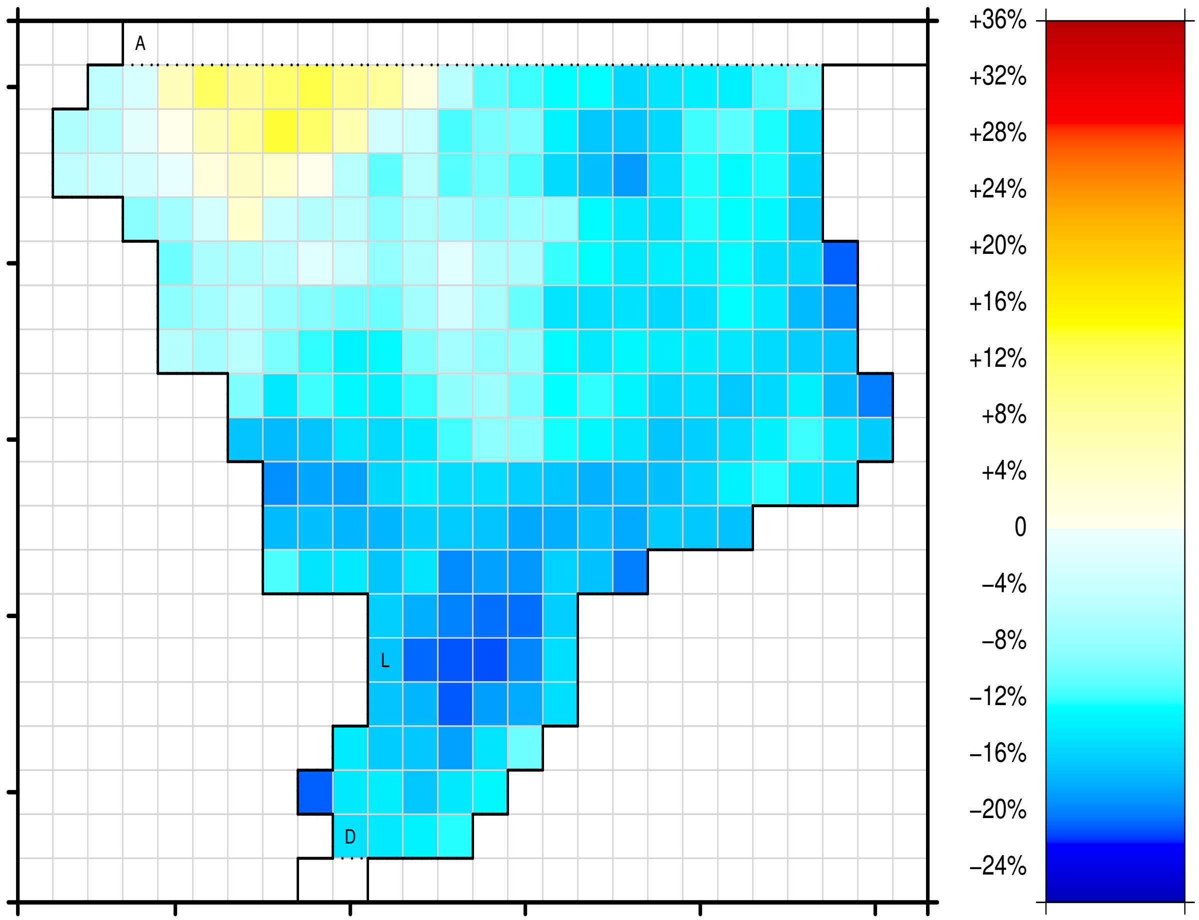
50y surge return levels: model output for Dover



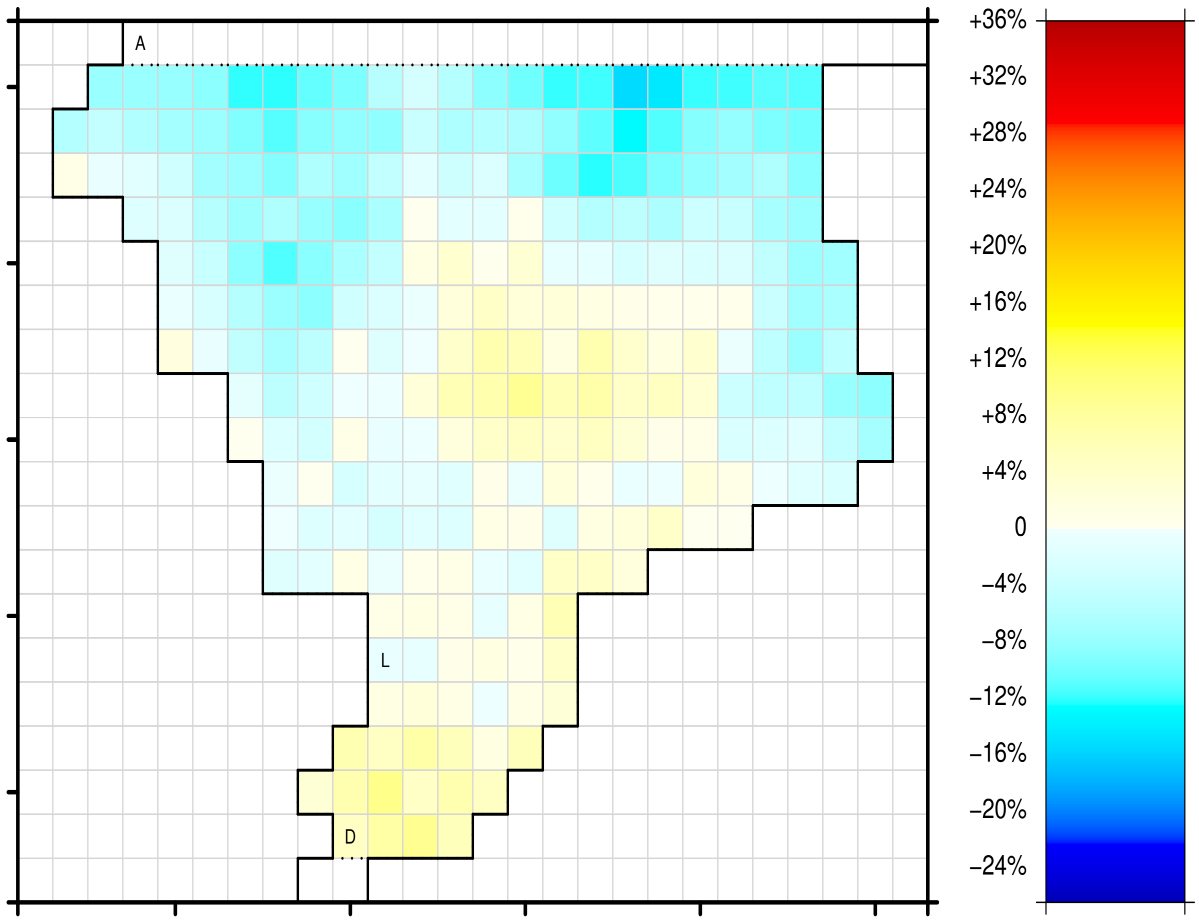
50y surge return levels: observational data for Dover



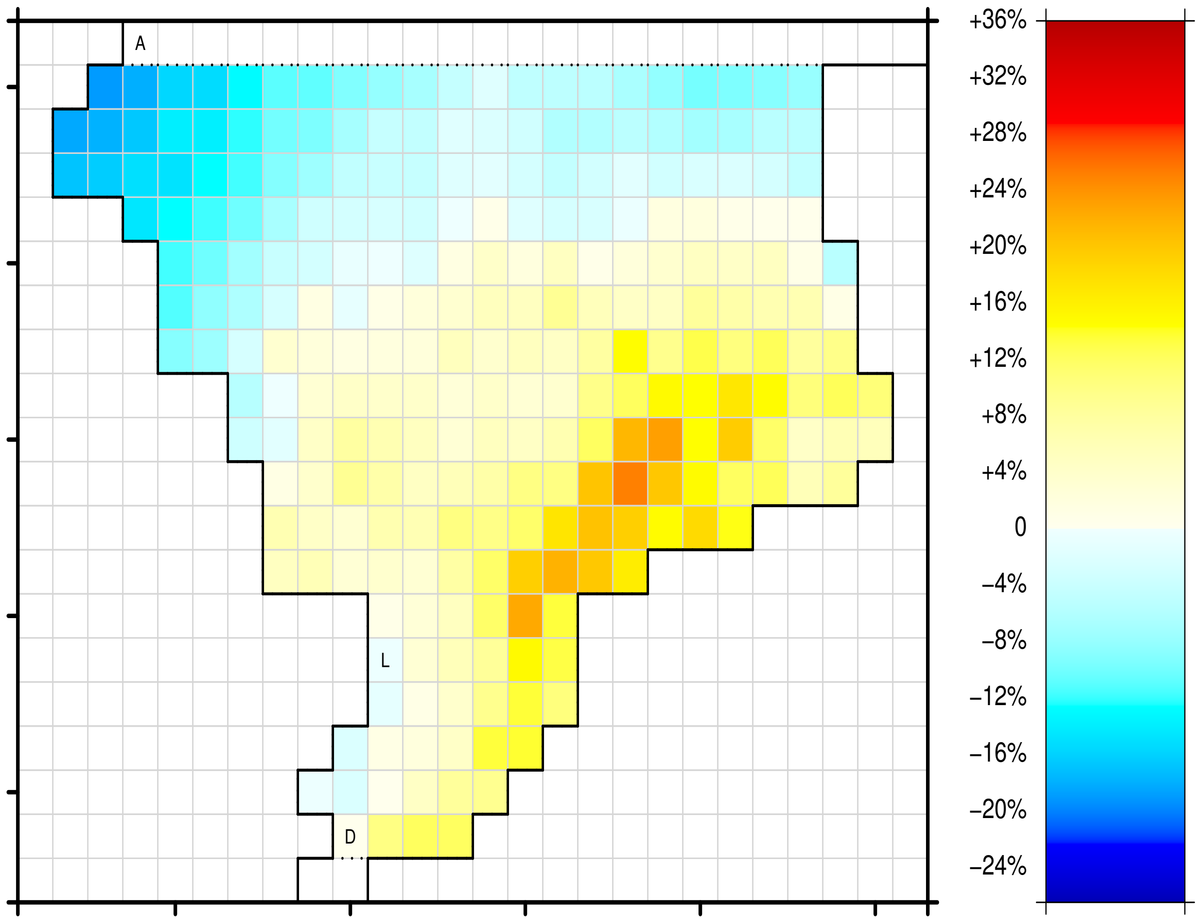
Smoothed variations in 50y return levels: 1955



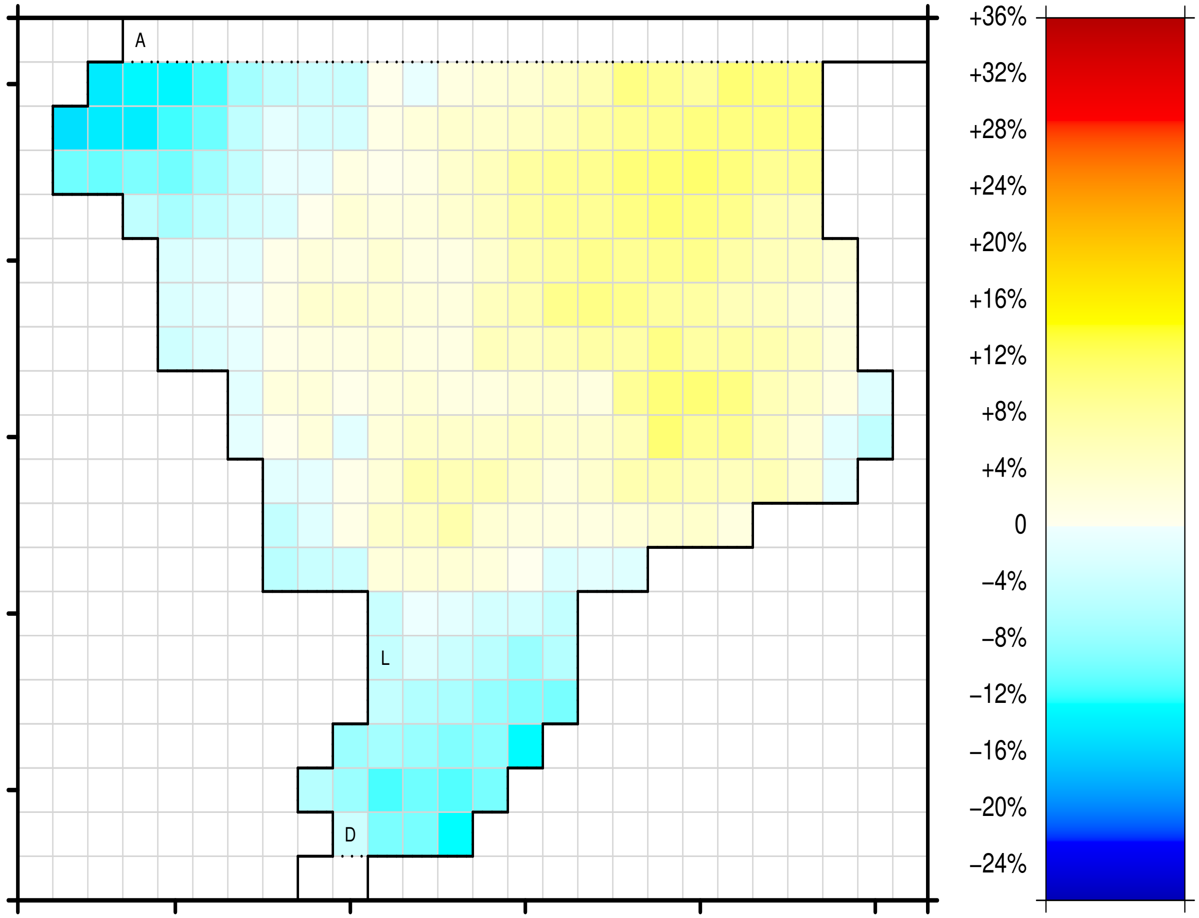
Smoothed variations in 50y return levels: 1962



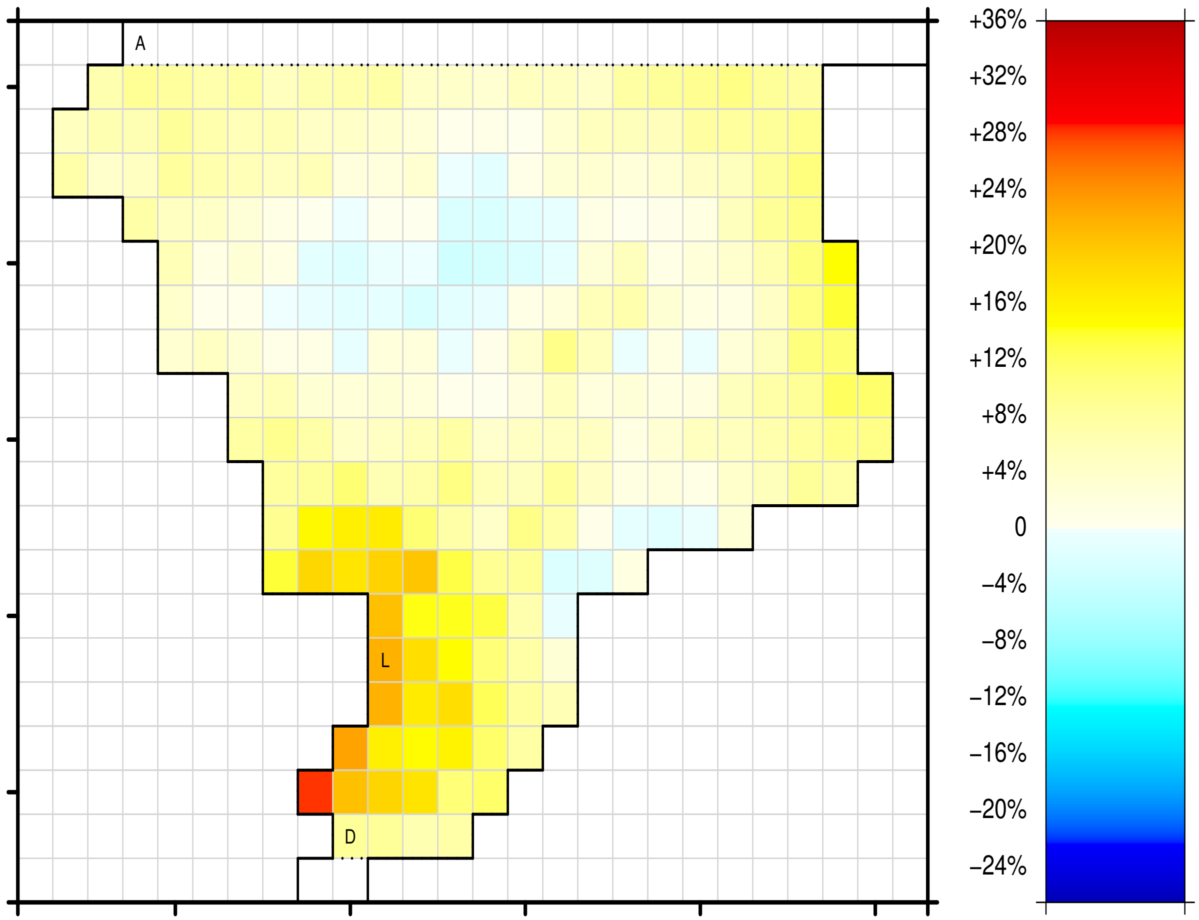
Smoothed variations in 50y return levels: 1975



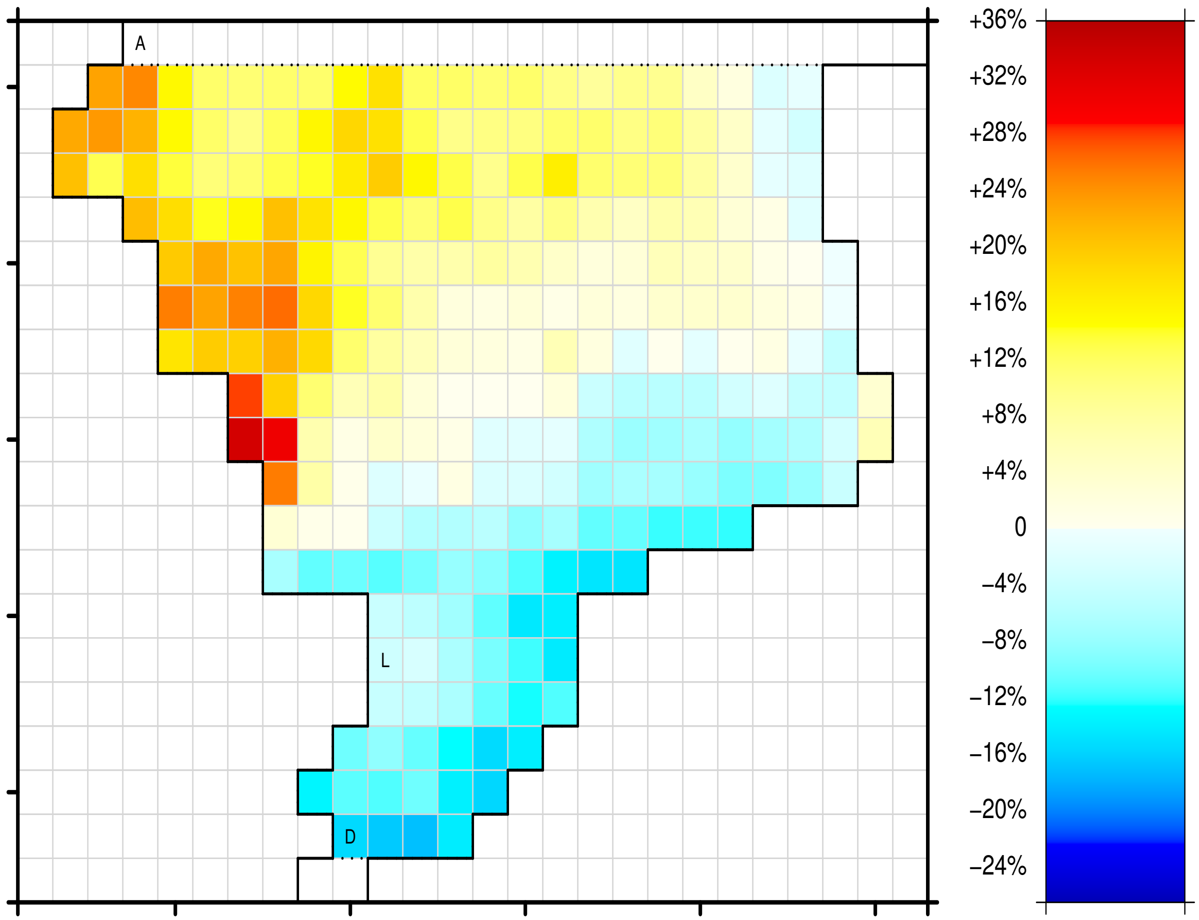
Smoothed variations in 50y return levels: 1982



Smoothed variations in 50y return levels: 1990



Smoothed variations in 50y return levels: 2000



- **Summary of findings**:

  NE: Fair evidence for increasing trends

  in magnitude & frequency of storm surges

  West: weak evidence for increasing trends

  South: dominated by decadal variability

  Similar findings if the NAO signal is accounted for

  Similar findings for surges on high tides

- **Spatio-temporal smoothing**:

  Reduce uncertainty by incorporating spatial information

  Need to ensure we smooth *only* the temporal trends

  Achieve this through a reparameterisation:

$$\bar{\mu}_{ij} = \mu_{ij}/\hat{\mu}_i$$

$$\bar{\sigma}_{ij} = \sigma_{ij}/\hat{\sigma}_i$$

  Use local likelihood to simultaneously smooth

  the parameters $\bar{\mu}_{ij}$, $\bar{\sigma}_{ij}$ and ξ_{ij} over space & time

4 : Wider statistical context

- **Applications of EVT**:

  Hydrology & oceanography (Katz *et al.*, 2002)

  Climatology & atmospheric chemistry (Smith, 1990)

  Engineering & telecommunications

  Finance (Chavez-Demoulin and Embrechts, 2004)

  Genetics (e.g. sequence alignment: BLAST)

  Ecology: disturbance (Katz *et al.*, 2005) & dispersal

- **Research: statistical modelling & inference**:

  Bayesian methods & MCMC

  Automatic threshold selection

  Nonparametric modelling of trends

  Markov models for time series extremes

- **Research: mathematical theory**:

  Characterising extremal dependence

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